# ACCURATE CALCULATION OF THE CAPACITANCE MATRIX OF A MULTICONDUCTOR TRANSMISSIONLINE IN A MULTILAYERED DIELECTRIC MEDIUM

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ABSTRACT

An integral equation method for the calculation of capacitance and inductance matrices is presented. The method is suited for multiconductor transmission lines embedded in a multilayered dielectric medium on top of a ground plane. Conductors of arbitrary polygonal cross-section can be handled, as well as infinitely thin conductors. The method is new in two respects. The kernel of the integral equation is the space domain Green's function of the layered medium. The accuracy of the solution is improved by using basis functions which exactly model the singular behaviour of the charge density in the neighbourhood of a conductor edge. Numerical examples show the accuracy of the calculations and the complexity of the configurations that can be treated.

## 1. INTRODUCTION

In high speed digital design, good modeling of the interconnections becomes increasingly important. In the quasi-TEM approximation a bus is completely described by its capacitance and inductance matrices. In this paper, a new method is presented for the numerical calculation of the capacitance and inductance matrices of complicated bus structures embedded in a multilayered dielectric on top of a ground plane.

We present an integral equation method which uses the Green's function of a layered dielectric on top of a ground plane. The number of dielectric layers is arbitrary. The Green's function, although in a first step calculated by spectral domain techniques, is transformed to the space domain. Conductors of arbitrary polygonal cross-sections can be treated, as well as infinitely thin strips. The integral equation is solved by the method of moments in conjunction with point-matching. The unknown surface charge density on the conductors is expanded in basis functions which accurately model the singular behaviour of the charge density in the neighbourhood of an edge. As we use the Green's function of the layered medium, no extra polarisation charges at the boundaries between the layers have to be taken into account as was the case in [1].

## 2. OUTLINE OF THE METHOD

Figure 1 shows the general geometry of the problem. A conductor with circular cross-section can be approximated by a regular polygon. The integral equation method is used to calculate the capacitance matrix

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$\underline{C}$ of the bus. The inductance matrix $\underline{L}$ is derived from the vacuum capacitance matrix $\underline{C}_v$, which is the capacitance matrix of the same bus structure, but with all dielectrics replaced by air, using the simple formula $\underline{L}.\underline{C}_v = \epsilon_o \mu_o$. The relevant integral equation is of the form:

$$\int_S \rho(\bar{r}')G(\bar{r}|\bar{r}')dS(\bar{r}') = V(\bar{r}) \qquad (1)$$

where:

| | |
|---|---|
| S | = collection of all conductor surfaces |
| $\bar{r}$ | = observation point |
| $\bar{r}'$ | = variable integration point |
| $\rho(\bar{r}')$ | = charge distribution over the conductor surfaces |
| $G(\bar{r}\mid\bar{r}')$ | = spatial Green's function of the layered dielectric medium |

The Green's function of the layered dielectric medium is the solution of:

$$\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} = \frac{-\delta(x,y-y')}{\epsilon_o \epsilon_r} \qquad (2)$$

where $\epsilon_r$ takes a constant value within each layer.

In order to determine G, a Fourier transformation with respect to x is introduced and G is first determined in the spatial Fourier domain. Inverse Fourier transformation leads to the spatial Green's function. Special care is taken to accurately determine this inverse transformation when source and observation point are closely spaced together. This in term allows a correct evaluation of the so-called self-patch contributions to (1).

The unknown charge distribution $\rho(\bar{r}')$ is expanded in basis functions:

$$\rho(\bar{r}') = \sum_{i=1}^{N} x_i \, f_i(\bar{r}') \qquad (3)$$

Each side of a polygon is divided into a number of elementary intervals. In the inner intervals, such as AB on fig. 2, linear basis functions are used:

$$\rho(t) = x_i \,(1 - t) + x_{i+1}\, t \qquad 0 \le t \le 1 \qquad (4)$$

where t is proportional with the arc length along an elementary interval. The coefficients $x_i$ and $x_{i+1}$ are the unknown coefficients of the basis functions and correspond with the charge densities in the endpoints of the interval.

In the outer intervals near the edges, such as EF, the first terms of a series expansion which accurately models the singular behaviour of the charge distribution in the neighbourhood of an edge are used [2]. In this case, the charge distribution takes the form

$$\rho(t) = x_1 \, t^{\nu-1} + x_2 \, t^{\nu} \qquad \nu > 0 \; ; \; 0 \le t \le 1 \qquad (5)$$

where t is proportional to the distance to the edge. The sum $x_1 + x_2$ corresponds to the charge density in the non-singular end-point of the interval. The value of $\nu$ is obtained from a formula which was derived by Meixner [2]

$$\frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2} = \frac{\sin\nu\phi_2}{\sin\nu(2\phi_1-\phi_2)} \qquad (6)$$

The meaning of the quantities $\epsilon_1$, $\epsilon_2$, $\phi_1$ and $\phi_2$ is made clear in Figure 2. Application of the point matching technique which consists in satisfying (1) in a limited number of points in the center or at the endpoints of each elementary interval reduces (1) to a set of linear algebraic equations. This set is solved by a least squares method.

## 3. NUMERICAL EXAMPLES

### a. Thin microstrip line

Analysis of the simple geometry of Figure 3 is used to illustrate the accuracy of the method. For a number of ratios W/H, where W is the stripwidth and H is the substrate thickness, the results of the method described above are compared to the results of the numerical method of Wei et al. [1], who uses the free-space Green's function, to the results of the well-known formulas of Gupta [3] and to the results of more recent and more accurate formulas by Hammerstad and Jensen [4]. The characteristic impedances that result from the different calculation methods are shown in Table 1. The results of Wei et al. are obtained by using 12 subsections on the strip, 15 subsections at the dielectric interface between -2W and -W/2, and another 15 subsections at the dielectric interface between W/2 and 2W. Our results are obtained by using 20 subsections on the strip. This required about 15 seconds CPU-time per value of the ratio W/H on a VAX 3200 workstation.

### b. Circular wires in a three-layered dielectric medium

Figure 4 shows a typical discrete wiring geometry [5]. The circular cross-sections are approximated by regular octagons. Each of the sides of the octagons is divided into 4 subsections, which leads to a total of 64 subsections. Table 2 shows the results of the calculations, which took 5 minutes and 27 seconds of CPU time on a VAX 3200 workstation.


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| W/H | Our results | Gupta | Wei et al. | Hammerstad |
|---|---|---|---|---|
| 0.4 | 90.3204 | 90.1907 | 92.2785 | 90.3339 |
| 0.7 | 72.7372 | 72.6731 | 73.9626 | 72.7516 |
| 1.0 | 61.8422 | 61.5907 | 62.8109 | 61.8397 |
| 2.0 | 42.2676 | 42.3945 | 42.9980 | 42.2600 |
| 4.0 | 26.4429 | 26.5168 | 26.9709 | 26.4593 |
| 10. | 12.7132 | 12.7164 | 12.9961 | 12.7198 |

Table 1: calculated characteristic impedances (Ω)

$$\underline{C} = \begin{vmatrix} 137.8 & -59.20 \\ -59.20 & 137.8 \end{vmatrix} \quad (\text{pF/m})$$

$$\underline{C}_V = \begin{vmatrix} 34.33 & -13.59 \\ -13.59 & 34.33 \end{vmatrix} \quad (\text{pF/m})$$

$$\underline{L} = \begin{vmatrix} 384.3 & 152.1 \\ 152.1 & 384.3 \end{vmatrix} \quad (\text{nH/m})$$

Table 2:
Capacitance, vacuum-capacitance and inductance matrices of the geometry of Figure 4.

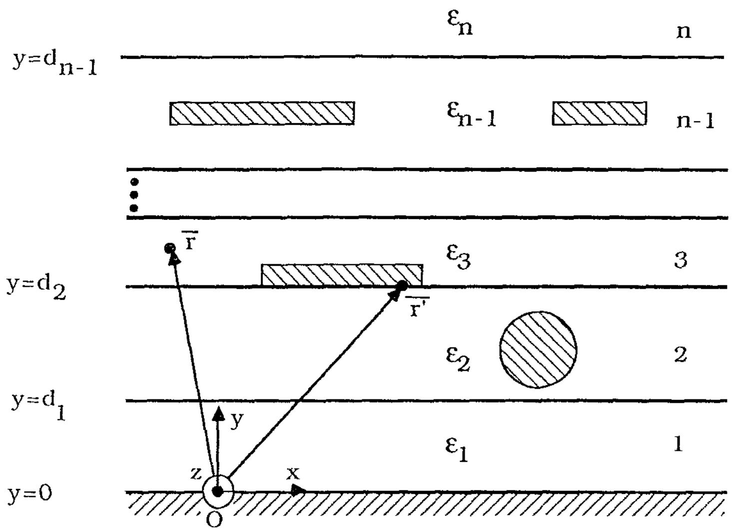


Figure 1: General geometry of the problem

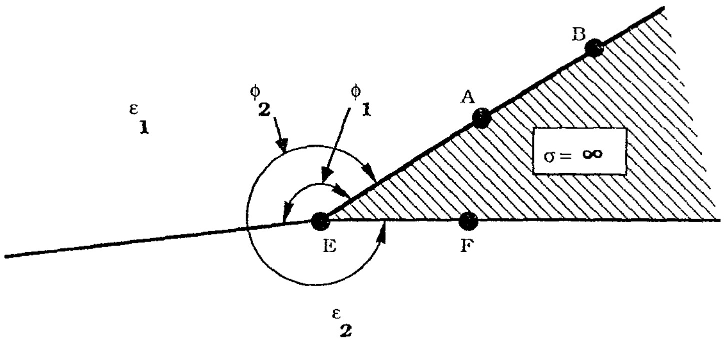


Figure 2: Conducting wedge at the interface between two dielectrics

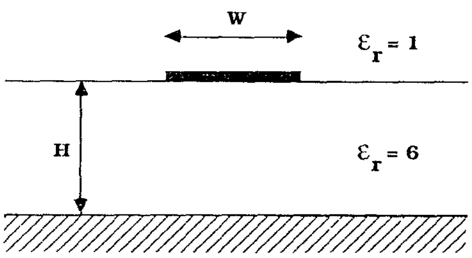


Figure 3: Thin microstrip line

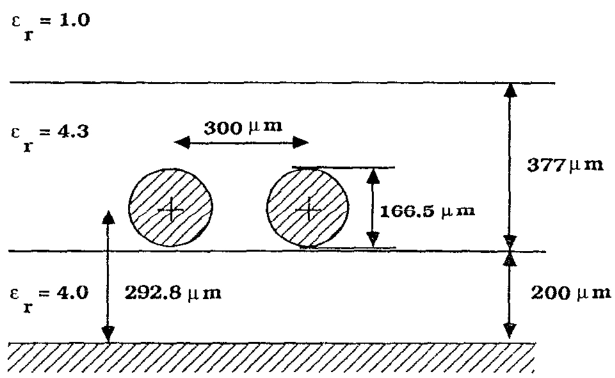


Figure 4: Geometry of example b